

# 1 Module Documentation

## 1.1 Weighted Constraint Satisfaction Problem file format (wcsp)

It is a text format composed of a list of numerical and string terms separated by spaces. Instead of using names for making reference to variables, variable indexes are employed. The same for domain values. All indexes start at zero.

Cost functions can be defined in intention (see below) or in extension, by their list of tuples. A default cost value is defined per function in order to reduce the size of the list. Only tuples with a different cost value should be given (not mandatory). All the cost values must be positive. The arity of a cost function in extension may be equal to zero. In this case, there is no tuples and the default cost value is added to the cost of any solution. This can be used to represent a global lower bound constant of the problem.

The wcsp file format is composed of three parts: a problem header, the list of variable domain sizes, and the list of cost functions.

- Header definition for a given problem:

```
<Problem name>
<Number of variables (N)>
<Maximum domain size>
<Number of cost functions>
<Initial global upper bound of the problem (UB)>
```

The goal is to find an assignment of all the variables with minimum total cost, strictly lower than UB. Tuples with a cost greater than or equal to UB are forbidden (hard constraint).

- Definition of domain sizes

```
<Domain size of variable with index 0>
...
<Domain size of variable with index N - 1>
```

### Note

domain values range from zero to *size-1*  
a negative domain size is interpreted as a variable with an interval domain in  $[0, -size - 1]$

### Warning

variables with interval domains are restricted to arithmetic and disjunctive cost functions in intention (see below)

- General definition of cost functions

- Definition of a cost function in extension

```
<Arity of the cost function>
<Index of the first variable in the scope of the cost function>
...
<Index of the last variable in the scope of the cost function>
<Default cost value>
<Number of tuples with a cost different than the default cost>
```

followed by for every tuple with a cost different than the default cost:

```
<Index of the value assigned to the first variable in the scope>
...
<Index of the value assigned to the last variable in the scope>
<Cost of the tuple>
```

**Note**

Shared cost function: A cost function in extension can be shared by several cost functions with the same arity (and same domain sizes) but different scopes. In order to do that, the cost function to be shared must start by a negative scope size. Each shared cost function implicitly receives an occurrence number starting from 1 and incremented at each new shared definition. New cost functions in extension can reuse some previously defined shared cost functions in extension by using a negative number of tuples representing the occurrence number of the desired shared cost function. Note that default costs should be the same in the shared and new cost functions. Here is an example of 4 variables with domain size 4 and one AllDifferent hard constraint decomposed into 6 binary constraints.

- Shared CF used inside a small example in wcsp format:

```
AllDifferentDecomposedIntoBinaryConstraints 4 4 6 1
4 4 4 4
-2 0 1 0 4
0 0 1
1 1 1
2 2 1
3 3 1
2 0 2 0 -1
2 0 3 0 -1
2 1 2 0 -1
2 1 3 0 -1
2 2 3 0 -1
```

- Definition of a cost function in intension by replacing the default cost value by -1 and by giving its keyword name and its K parameters

```
<Arity of the cost function>
<Index of the first variable in the scope of the cost function>
...
<Index of the last variable in the scope of the cost function>
-1
<keyword>
<parameter1>
...
<parameterK>
```

Possible keywords of cost functions defined in intension followed by their specific parameters:

- $\geq$  *cst delta* to express soft binary constraint  $x \geq y + cst$  with associated cost function  $max((y + cst - x \leq delta)?(y + cst - x) : UB, 0)$
- $>$  *cst delta* to express soft binary constraint  $x > y + cst$  with associated cost function  $max((y + cst + 1 - x \leq delta)?(y + cst + 1 - x) : UB, 0)$
- $\leq$  *cst delta* to express soft binary constraint  $x \leq y + cst$  with associated cost function  $max((x - cst - y \leq delta)?(x - cst - y) : UB, 0)$
- $<$  *cst delta* to express soft binary constraint  $x < y + cst$  with associated cost function  $max((x - cst + 1 - y \leq delta)?(x - cst + 1 - y) : UB, 0)$
- $=$  *cst delta* to express soft binary constraint  $x = y + cst$  with associated cost function  $(|y + cst - x| \leq delta)?|y + cst - x| : UB$
- *disj cstx csty penalty* to express soft binary disjunctive constraint  $x \geq y + cst \vee y \geq x + cst$  with associated cost function  $(x \geq y + cst \vee y \geq x + cst)?0 : penalty$
- *sdisj cstx csty xinfy yinfy costx costy* to express a special disjunctive constraint with three implicit hard constraints  $x \leq xinfy$  and  $y \leq yinfy$  and  $x < xinfy \wedge y < yinfy \Rightarrow (x \geq y + cst \vee y \geq x + cst)$  and an additional cost function  $((x = xinfy)?costx : 0) + ((y = yinfy)?costy : 0)$
- Global cost functions using a dedicated propagator:
  - clique 1 (*nb\_values (value)\**)\* to express a hard clique cut to restrict the number of variables taking their value into a given set of values (per variable) to at most 1 occurrence for all the variables (warning! it assumes also a clique of binary constraints already exists to forbid any two variables using both the restricted values)

- Global cost functions using a flow-based propagator:
  - `salldiff var|dec|decbi cost` to express a soft alldifferent constraint with either variable-based (`var` keyword) or decomposition-based (`dec` and `decbi` keywords) cost semantic with a given `cost` per violation (`decbi` decomposes into a binary cost function complete network)
  - `sgcc var|dec|wdec cost nb_values (value lower_bound upper_bound (shortage_weight excess_weight)?)*` to express a soft global cardinality constraint with either variable-based (`var` keyword) or decomposition-based (`dec` keyword) cost semantic with a given `cost` per violation and for each value its lower and upper bound (if `wdec` then violation cost depends on each value shortage or excess weights)
  - `ssame cost list_size1 list_size2 (variable_index)* (variable_index)*` to express a permutation constraint on two lists of variables of equal size (implicit variable-based cost semantic)
  - `sregular var|edit cost nb_states nb_initial_states (state)* nb_final_states (state)* nb_transitions (start_state symbol_value end_state)*` to express a soft regular constraint with either variable-based (`var` keyword) or edit distance-based (`edit` keyword) cost semantic with a given `cost` per violation followed by the definition of a deterministic finite automaton with number of states, list of initial and final states, and list of state transitions where symbols are domain values
- Global cost functions using a dynamic programming DAG-based propagator:
  - `sregulardp var cost nb_states nb_initial_states (state)* nb_final_states (state)* nb_transitions (start_state symbol_value end_state)*` to express a soft regular constraint with a variable-based (`var` keyword) cost semantic with a given `cost` per violation followed by the definition of a deterministic finite automaton with number of states, list of initial and final states, and list of state transitions where symbols are domain values
  - `sgrammar|sgrammardp var|weight cost nb_symbols nb_values start_symbol nb_rules ((0 terminal_symbol_value)|(1 nonterminal_in nonterminal_out_left nonterminal_out_right)|(2 terminal_symbol_value weight)|(3 nonterminal_in nonterminal_out_left nonterminal_out_right weight))*` to express a soft/weighted grammar in Chomsky normal form
  - `samong|samongdp var cost lower_bound upper_bound nb_values (value)*` to express a soft among constraint to restrict the number of variables taking their value into a given set of values
  - `salldifddp var cost` to express a soft alldifferent constraint with variable-based (`var` keyword) cost semantic with a given `cost` per violation (decomposes into `samongdp` cost functions)
  - `sgccdp var cost nb_values (value lower_bound upper_bound)*` to express a soft global cardinality constraint with variable-based (`var` keyword) cost semantic with a given `cost` per violation and for each value its lower and upper bound (decomposes into `samongdp` cost functions)
  - `max|smaxdp defCost nbtuples (variable value cost)*` to express a weighted max cost function to find the maximum cost over a set of unary cost functions associated to a set of variables (by default, `defCost` if unspecified)
  - `MST|smstdp` to express a spanning tree hard constraint where each variable is assigned to its parent variable index in order to build a spanning tree (the root being assigned to itself)
- Global cost functions using a cost function network-based propagator:
  - `wregular nb_states nb_initial_states (state and cost)* nb_final_states (state and cost)* nb_transitions (start_state symbol_value end_state cost)*` to express a weighted regular constraint with weights on initial states, final states, and transitions, followed by the definition of a deterministic finite automaton with number of states, list of initial and final states with their costs, and list of weighted state transitions where symbols are domain values
  - `walldiff hard|lin|quad cost` to express a soft alldifferent constraint as a set of `wamong` hard constraint (`hard` keyword) or decomposition-based (`lin` and `quad` keywords) cost semantic with a given `cost` per violation
  - `wgcc hard|lin|quad cost nb_values (value lower_bound upper_bound)*` to express a soft global cardinality constraint as either a hard constraint (`hard` keyword) or with decomposition-based (`lin` and `quad` keyword) cost semantic with a given `cost` per violation and for each value its lower and upper bound
  - `wsame hard|lin|quad cost` to express a permutation constraint on two lists of variables of equal size (implicitly concatenated in the scope) using implicit decomposition-based cost semantic

- `wsamegcc hard|lin|quad cost nb_values (value lower_bound upper_bound)*` to express the combination of a soft global cardinality constraint and a permutation constraint
- `wamong hard|lin|quad cost nb_values (value)* lower_bound upper_bound` to express a soft among constraint to restrict the number of variables taking their value into a given set of values
- `wvamong hard cost nb_values (value)*` to express a hard among constraint to restrict the number of variables taking their value into a given set of values to be equal to the last variable in the scope
- `woverlap hard|lin|quad cost comparator righthandside` overlaps between two sequences of variables  $X_i$ ,  $Y_j$  (i.e. set the fact that  $X_i$  and  $Y_j$  take the same value (not equal to zero))
- `wsum hard|lin|quad cost comparator righthandside` to express a soft sum constraint with unit coefficients to test if the sum of a set of variables matches with a given comparator and right-hand-side value
- `wvarsum hard cost comparator` to express a hard sum constraint to restrict the sum to be *comparator* to the value of the last variable in the scope

Let us note  $\langle \rangle$  the comparator,  $K$  the right-hand-side value associated to the comparator, and  $\text{Sum}$  the result of the sum over the variables. For each comparator, the gap is defined according to the distance as follows:

- \* if  $\langle \rangle$  is `==` :  $\text{gap} = \text{abs}(K - \text{Sum})$
- \* if  $\langle \rangle$  is `<=` :  $\text{gap} = \max(0, \text{Sum} - K)$
- \* if  $\langle \rangle$  is `<` :  $\text{gap} = \max(0, \text{Sum} - K - 1)$
- \* if  $\langle \rangle$  is `!=` :  $\text{gap} = 1$  if  $\text{Sum} \neq K$  and  $\text{gap} = 0$  otherwise
- \* if  $\langle \rangle$  is `>` :  $\text{gap} = \max(0, K - \text{Sum} + 1)$ ;
- \* if  $\langle \rangle$  is `>=` :  $\text{gap} = \max(0, K - \text{Sum})$ ;

### Warning

The decomposition of `wsum` and `wvarsum` may use an exponential size (sum of domain sizes).  
`list_size1` and `list_size2` must be equal in `ssame`.  
 Cost functions defined in intention cannot be shared.

### Note

More about network-based global cost functions can be found here <https://metivier.users.greyc.fr/decomposable/>

### Examples:

- quadratic cost function  $x_0 * x_1$  in extension with variable domains  $\{0, 1\}$  (equivalent to a soft clause  $\neg x_0 \vee \neg x_1$ ):

```
2 0 1 0 1 1 1 1
```

- simple arithmetic hard constraint  $x_1 < x_2$ :

```
2 1 2 -1 < 0 0
```

- hard temporal disjunction  $x_1 \geq x_2 + 2 \vee x_2 \geq x_1 + 1$ :

```
2 1 2 -1 disj 1 2 UB
```

- clique cut ( $\{x_0, x_1, x_2, x_3\}$ ) on Boolean variables such that value 1 is used at most once:

```
4 0 1 2 3 -1 clique 1 1 1 1 1 1 1 1
```

- `soft_alldifferent`( $\{x_0, x_1, x_2, x_3\}$ ):

```
4 0 1 2 3 -1 salldiff var 1
```

- `soft_gcc({x1,x2,x3,x4})` with each value  $v$  from 1 to 4 only appearing at least  $v-1$  and at most  $v+1$  times:

```
4 1 2 3 4 -1 sgcc var 1 4 1 0 2 2 1 3 3 2 4 4 3 5
```

- `soft_same({x0,x1,x2,x3},{x4,x5,x6,x7})`:

```
8 0 1 2 3 4 5 6 7 -1 ssame 1 4 4 0 1 2 3 4 5 6 7
```

- `soft_regular({x1,x2,x3,x4})` with DFA  $(3^*)+(4^*)$ :

```
4 1 2 3 4 -1 sregular var 1 2 1 0 2 0 1 3 0 3 0 0 4 1 1 4 1
```

- `soft_grammar({x0,x1,x2,x3})` with hard cost (1000) producing well-formed parenthesis expressions:

```
4 0 1 2 3 -1 sgrammardp var 1000 4 2 0 6 1 0 0 0 1 0 1 2 1 0 1 3 1 2 0 3 0 1 0 0 3 1
```

- `soft_among({x1,x2,x3,x4})` with hard cost (1000) if  $\sum_{i=1}^4 (x_i \in \{1,2\}) < 1$  or  $\sum_{i=1}^4 (x_i \in \{1,2\}) > 3$ :

```
4 1 2 3 4 -1 samongdp var 1000 1 3 2 1 2
```

- `soft_max({x0,x1,x2,x3})` with cost equal to  $\max_{i=0}^3 ((x_i \neq i) ? 1000 : (4 - i))$ :

```
4 0 1 2 3 -1 smaxdp 1000 4 0 0 4 1 1 3 2 2 2 3 3 1
```

- `wregular({x0,x1,x2,x3})` with DFA  $0(10)^*2^*$ :

```
4 0 1 2 3 -1 wregular 3 1 0 0 1 2 0 9 0 0 1 0 0 1 1 1 0 2 1 1 1 1 0 0 1 0 0 1 1 2 0 1 1 2 2 0 1 0 2 1 1 1 2
1
```

- `wamong ({x1,x2,x3,x4})` with hard cost (1000) if  $\sum_{i=1}^4 (x_i \in \{1,2\}) < 1$  or  $\sum_{i=1}^4 (x_i \in \{1,2\}) > 3$ :

```
4 1 2 3 4 -1 wamong hard 1000 2 1 2 1 3
```

- `wvamong ({x1,x2,x3,x4})` with hard cost (1000) if  $\sum_{i=1}^3 (x_i \in \{1,2\}) \neq x_4$ :

```
4 1 2 3 4 -1 wvamong hard 1000 2 1 2
```

- `woverlap({x1,x2,x3,x4})` with hard cost (1000) if  $\sum_{i=1}^2 (x_i = x_{i+2}) \geq 1$ :

```
4 1 2 3 4 -1 woverlap hard 1000 < 1
```

- `wsum ({x1,x2,x3,x4})` with hard cost (1000) if  $\sum_{i=1}^4 (x_i) \neq 4$ :

```
4 1 2 3 4 -1 wsum hard 1000 == 4
```

- `wvarsum ({x1,x2,x3,x4})` with hard cost (1000) if  $\sum_{i=1}^3 (x_i) \neq x_4$ :

```
4 1 2 3 4 -1 wvarsum hard 1000 ==
```

Latin Square 4 x 4 crisp CSP example in wvsp format:

```
latin4 16 4 8 1
4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4 0 1 2 3 -1 salldiff var 1
4 4 5 6 7 -1 salldiff var 1
4 8 9 10 11 -1 salldiff var 1
4 12 13 14 15 -1 salldiff var 1
4 0 4 8 12 -1 salldiff var 1
4 1 5 9 13 -1 salldiff var 1
4 2 6 10 14 -1 salldiff var 1
4 3 7 11 15 -1 salldiff var 1
```

4-queens binary weighted CSP example with random unary costs in wcp format:

```
4-WQUEENS 4 4 10 5
4 4 4 4
2 0 1 0 10
0 0 5
0 1 5
1 0 5
1 1 5
1 2 5
2 1 5
2 2 5
2 3 5
3 2 5
3 3 5
2 0 2 0 8
0 0 5
0 2 5
1 1 5
1 3 5
2 0 5
2 2 5
3 1 5
3 3 5
2 0 3 0 6
0 0 5
0 3 5
1 1 5
2 2 5
3 0 5
3 3 5
2 1 2 0 10
0 0 5
0 1 5
1 0 5
1 1 5
1 2 5
2 1 5
2 2 5
2 3 5
3 2 5
3 3 5
2 1 3 0 8
0 0 5
0 2 5
1 1 5
1 3 5
2 0 5
2 2 5
3 1 5
3 3 5
2 2 3 0 10
0 0 5
0 1 5
1 0 5
1 1 5
1 2 5
2 1 5
2 2 5
2 3 5
3 2 5
3 3 5
1 0 0 2
1 1
3 1
1 1 0 2
1 1
2 1
1 2 0 2
1 1
2 1
1 3 0 2
0 1
2 1
```